

# Lesson 5: Linear Patterns in Data



## Selected Content Standards

### **Benchmarks Addressed:**

- P-1-M** *Describing, extending, analyzing, and creating a wide variety of numerical, geometrical, and statistical patterns (e.g., skip counting of rational numbers and simple exponential number patterns)*
- D-2-M** *Analyzing, interpreting, evaluating, drawing inferences, and making estimations, predictions, decisions, and convincing arguments based on organized data (e.g., analyze data using concepts of mean, median, mode, range, random samples, sample size, bias, and data extremes)*
- A-3-M** *Representing situations and number patterns with tables, graphs, and verbal and written statements, while exploring the relationships among these representations (e.g., multiple representations for the same situation)*
- P-1-H** *Modeling the concepts of variables, functions, and relations as they occur in the real world and using the appropriate notation and terminology*

### **GLEs Addressed:**

#### Grade 8

13. Switch between functions represented as tables, equations, graphs, and verbal representations, with and without technology (A-3-M) (P-2-M) (A-4-M)
38. Sketch and interpret a trend line (i.e., line of best fit) on a scatterplot (D-2-M) (A-4-M) (A-5-M)
39. Analyze and make predictions from discovered data patterns (D-2-M)
47. Represent the  $n$ th term in a pattern as a formula and test the representation (P-1-M) (P-2-M) (P-3-M) (A-5-M)

#### Grade 9

39. Analyze real-life relationships that can be modeled by linear functions (P-1-H) (P-5-H)

### **Lesson Focus**

This lesson is intended as an extension of the lesson on graphing linear equations and as an introduction to connecting patterns in tables and graphs to algebraic representations. It includes the following:

- Use algebraic equations to represent patterns found in tables and charts.
- Determine patterns from instances given, complete charts on the information, and write equations from those charts.
- Interpolate and extrapolate information from graphs.

### **GEE 21 Connection**

The skills that will be addressed in this lesson include the following:

- Analyze linear patterns in data and represent them algebraically.

- Use information presented in a variety of graphs, charts, and tables to problem solve.

### Translating Content Standards into Instruction

- A. We want the students to be able to analyze information found in table and charts and represent that information algebraically. Begin by getting the students to examine a simple chart from **Teacher Blackline #1, Part A** and determine the patterns that exist. How are  $x$  and  $y$  related?

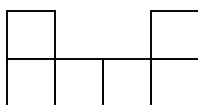
$x$	$y$
0	15
1	17
2	19
3	21
4	23

At the beginning, leave the discussion open for any and all observations. One general observation of the relationship should be that  $y$  increases as  $x$  increases. Some specific observations should include the start up value for  $y$  is 15 and that as  $x$  increases by 1,  $y$  increases by 2. After noting all observations, ask the students to write an equation ( $y = 15 + 2x$ ) to describe the relationship between  $x$  and  $y$ . Discuss with the students how they can check their equation to be sure it accurately describes the patterns. We want students to recognize that all the ordered pairs from the chart should fit the equation.

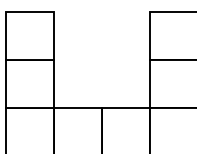
Ask students to determine some values of the relationship not given on the chart. What would  $y$  be if  $x = 10$ ? If  $x = 50$ ?

- B. Next we want the students to use equations to describe patterns. Have students examine each instance of the pattern shown on **Teacher Blackline #1, Part B** and fill in the chart.

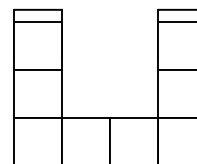
1<sup>st</sup> instance



2<sup>nd</sup> instance



3<sup>rd</sup> instance



Instance in Pattern	1	2	3	4	5		$n$
Number of Squares	6						

After the students have filled in the chart, discuss their observations. Ask the students to determine the equation that describes the relationship between the instance in the pattern and the number of squares. You may first want to have students determine how many squares are added in each instance. Also have students determine the start up value for the number of squares.

Ask the students to use the equation to determine the number of squares needed for the 12<sup>th</sup> instance? For the 25<sup>th</sup> instance of the pattern?

- C. Have students discuss the meaning of the information in the chart below which is presented on **Teacher Blackline #2, Part C**.

<u>Year of Birth</u>	<u>Number of Years of Life Expectancy</u>
1920	54.1
1930	59.7
1940	62.9
1950	68.2
1960	69.7
1970	70.8
1980	73.7
1990	75.4

\*Source of life expectancy information – *The World Almanac 2001*

Ask the students if they see a pattern from the data. Ask the students if they can determine whether or not the data forms a linear pattern. Get the students to graph the data and examine the patterns in the graph. Discuss with the students how the data does not line up perfectly, but that there is a linear pattern. Discuss with the students the concept of **line of best fit**. (A single piece of uncooked spaghetti on the overhead works well to represent the line.)

We want the students to interpolate and extrapolate data. To interpolate information from the graph means the students will determine a value between two existing data points. One of the most meaningful points might be the year of their birth. For example, ask students to determine the life expectancy for a person born in 1987. Have the students examine the graph with the line of best fit and determine approximately what the  $y$ -value for the year 1987 would be. To extrapolate data from the graph means the

students will have to go above or below all the given data to determine a data value. Ask the students to determine the life expectancy of a person born in the year 2000. They will have to follow the line of best fit beyond the last data point to determine this value.

**Calculator Note:** Students can enter data points on their graphing calculators and have the calculators determine the line of best fit.

### Sources of Evidence about Student Learning

- A. The teacher should engage the students in discussions as they work through the examples on the teacher blacklines to determine if students are learning the concepts presented.
- B. Have the students work through the student worksheets, discuss their answers in small groups, and revise their own work. Also have students recognize the variety of correct ways that the problems were worked, by asking for examples and encouraging alternative methods of finding solutions.
- C. Have students gather their own data from experimentation or from a source such as *The World Almanac*, and determine if that data forms a linear pattern. Have students make predictions about data points that are not included in the data set.

### GEE 21 Connection

Sample items similar to what students might see on the GEE 21 test include:

(1) Which table matches the following rule?

**Multiply  $y$  by 2 and then subtract 1 to get  $x$ .**

a)

$y$	$X$
1	1
2	4
3	7

b)

$y$	$X$
1	1
2	4
3	5

c)

$y$	$X$
1	1
2	3
3	7

d)

<b>y</b>	<b>x</b>
1	1
2	3
3	5

(2) Which equation describes this table?

<b>R</b>	<b>S</b>
1	5
2	7
3	9
4	11

- a)  $S = R + 4$
- b)  $R = S - 7$
- c)  $S = 2R + 3$
- d)  $S = 6R - 1$

*Kentucky Released Multiple Choice Items, Grade 5, 1997-98*

(3) Use the chart below to answer this question.

<b>Time in Hours (<i>h</i>)</b>	<b>Price (<i>P</i>)</b>
1	\$10
2 ½	\$16
4	\$22
5	\$26
8	\$38

Each Saturday Bill and Carol rent a canoe. The chart above shows the amount of time of and the price paid for each rental. Which equation will help Bill and Carol determine the price of their next canoe rental?

- a)  $P = 10h$
- b)  $P = 10h + 6$
- c)  $P = 4h + 6$
- d)  $P = 6h + 4$

*Kentucky Released Multiple Choice Items, Grade 11, 1997-98*

(4) Which equation shows the relationship between  $x$  and  $y$  in the table?

<b>x</b>	2	6	8	12	14
<b>y</b>	3	27	39	63	75

- a)  $y = 9x - 6$
- b)  $y = 6x - 9$
- c)  $y = -6x - 9$
- d)  $y = -9x + 6$

*Massachusetts Grade 10 MCAS Re-Test Study Questions, 2000*

(5) The Pizza Palace's price list for plain pizzas is shown below

<b>Diameter</b>	<b>Cost</b>
10"	\$5.00
12"	\$7.20
14"	\$9.80
16"	\$12.80

Based on this information, what would a 20" pizza likely cost?

- a) \$10.00
- b) \$14.40
- c) \$14.80
- d) \$20.00

*Massachusetts Grade 10 MCAS Re-Test Study Questions, 1999*

(6) The cost  $c$  to rent a motorbike is given by the following formula:

$$c = 3h + 2 \quad \text{where } h \text{ is the number of hours.}$$

Fill in the table below.

<b>Time in hours (<math>h</math>)</b>	<b>Cost in \$ (<math>c</math>)</b>
1	
4	
	17

*NAEP*

### **Attributes of Student Work at the “Got-It” Level**

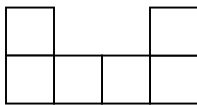
- A. When students are working through the examples, they should be able to show their understanding of the relationships between two variable quantities in class discussions.
- B. On the student worksheets, students should be able to examine charts with linear data and write an equation that shows the relationship between the quantities on the chart.
- C. On the student worksheets, students should be able to predict information not given on the charts and graphs using equations that fit the data.

A.

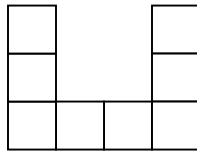
<b>x</b>	<b>y</b>
0	15
1	17
2	19
3	21
4	23

B.

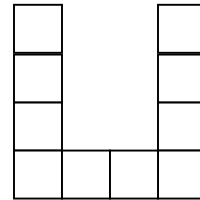
1<sup>st</sup> instance



2<sup>nd</sup> instance



3<sup>rd</sup> instance

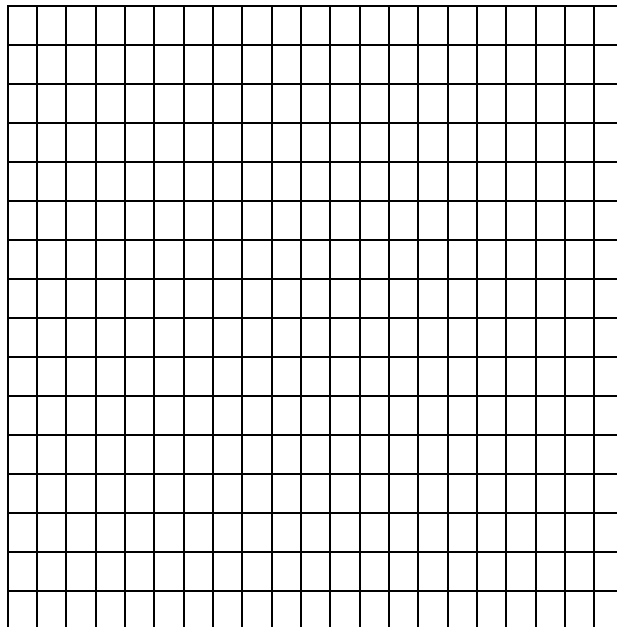


<b>Instance in Pattern</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>		<b>n</b>
<b>Number of Squares</b>	<b>6</b>						

C.

<b><u>Year of Birth</u></b>	<b>Number of Years of Life Expectancy</b>
<b>1920</b>	<b>54.1</b>
<b>1930</b>	<b>59.7</b>
<b>1940</b>	<b>62.9</b>
<b>1950</b>	<b>68.2</b>
<b>1960</b>	<b>69.7</b>
<b>1970</b>	<b>70.8</b>
<b>1980</b>	<b>73.7</b>
<b>1990</b>	<b>75.4</b>

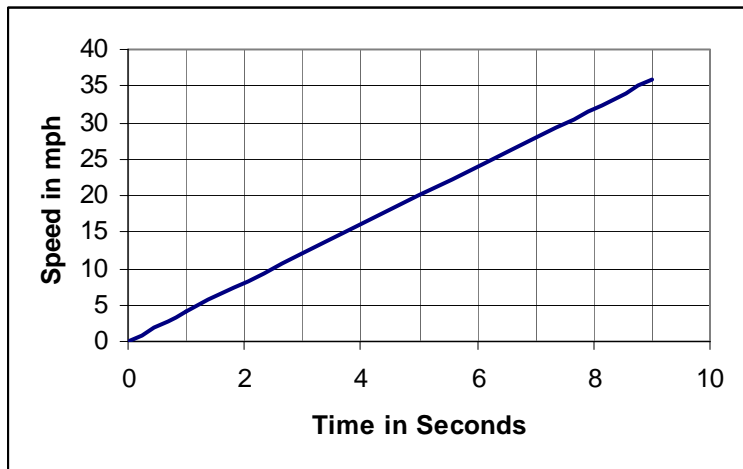
\*Source of life expectancy information -  
*The World Almanac 2001*



1) Does the equation  $s = 8r$ , describe the numbers in the chart? Explain.

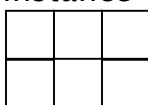
$r$	$s$
1	8
2	9
3	10
4	11

2) Use the graph below to find the speed when  $t = 5$  sec. Estimate the time when the speed is 30 mph.

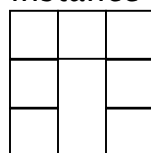


3) Examine the pattern below and complete the chart.

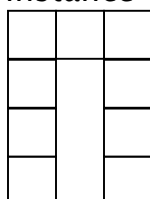
1<sup>st</sup> instance



2<sup>nd</sup> instance



3<sup>rd</sup> instance



instance	perimeter
1	12
2	
3	
4	
x	

Each time the instance increases by 1, the perimeter increases by how many?  
 What would be the equation to describe the perimeter  $p$  for the  $x$  instance?  
 What would the perimeter be on the 12<sup>th</sup> instance?

4) Examine the chart below that lists the parking charges for two different garages.

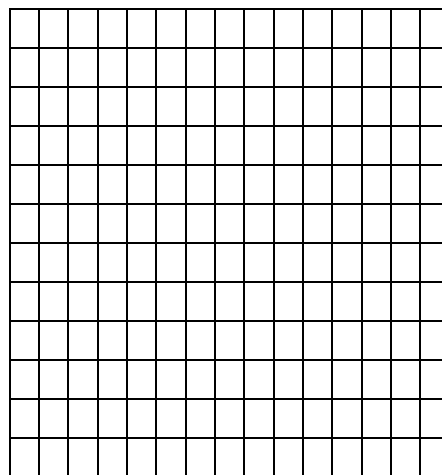
# of hours	Acme Park & Pay	Greg's Garage
1	\$0.75	\$1.20
2	\$1.00	\$1.30
3	\$1.25	\$1.40
4	\$1.50	\$1.50
5	\$1.75	\$1.60
6	\$2.00	\$1.70

- Which garage is cheaper?
- For how many hours of parking do the two garages charge the same amount?
- What would the graph look like when the two garages are charging the same amount?
- What is the charge per hour for parking in Greg's Garage?
- What is the equation for the total charge,  $c$ , for parking  $h$  hours in Acme Park & Pay? What is the equation for Greg's Garage?

5) The following chart shows the average height of U. S. children from ages 7 to 15.

Age	7	8	9	10	11	12	13	14	15
Height in cm	119.3	127.0	132.0	137.1	142.2	147.3	152.4	157.5	162.2

- Graph the relation on the grid provided.
- Use a ruler and draw a line of best fit.
- If the pattern continued, what would the average height of a 16 year old be?
- What would the average height of a child  $11 \frac{1}{2}$  years old?



Teacher Blackline #1

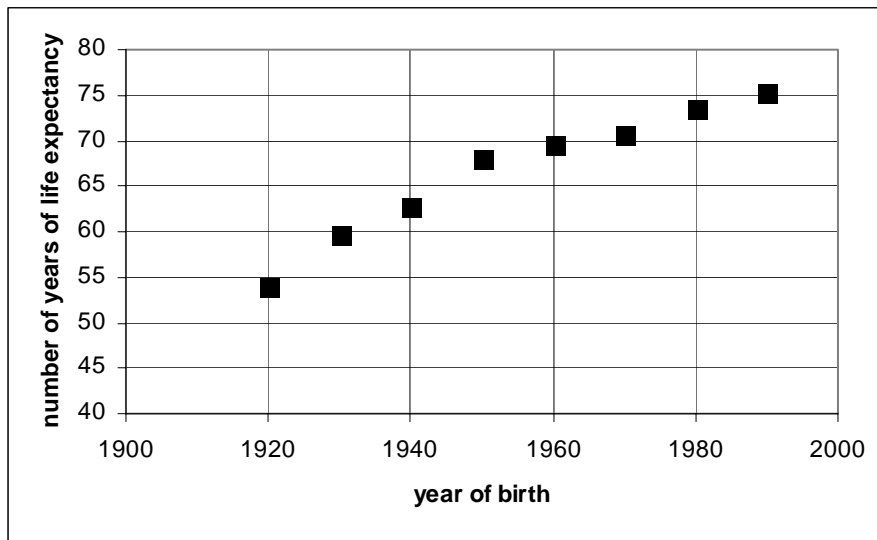
A.  $y = 15 + 2x$

B.

<b>Instance in Pattern</b>	1	2	3	4	5		n
<b>Number of Squares</b>	6	8	10	12	14		$2n+4$

Teacher Blackline #2

C.



Student Worksheet

1) The equation  $s = 8r$  only works for the first ordered pair. The equation that would fit all points in the chart is  $s = 7 + r$ .

2) When  $t = 5$  sec, the speed is 20 mph.  
When the speed is 30 mph, the time is 7.5 seconds.

3)

<b>instance</b>	<b>perimeter</b>
1	12
2	16
3	20
4	24
x	$4x + 8$

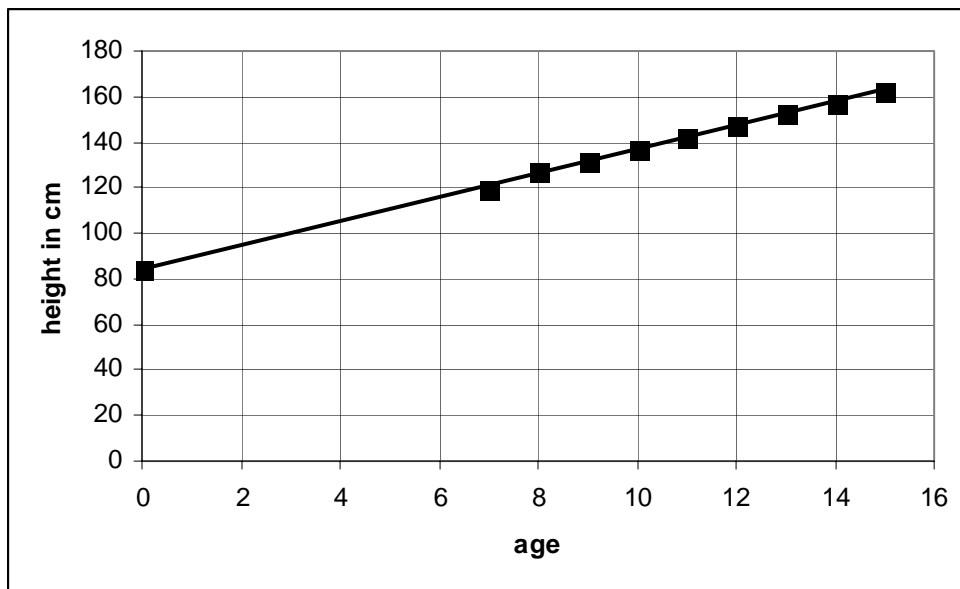
Each time the instance increases by 1, the perimeter increases by 4.  
 $p = 4x + 8$ . On the 12<sup>th</sup> instance, the perimeter would be 56.

## Student Worksheet – Algebra – Lesson 5

4)

- Acme Park & Pay is cheaper for the first 3 hours. After 4 hours, Greg's Garage is cheaper.
- The two garages charge the same amount for 4 hours of parking.
- When the two garages are charging the same amount, the two lines will intersect on the graph.
- Greg's Garage charges \$0.10 for an hour of parking.
- The equation for the total charge,  $c$ , for parking  $h$  hours in Acme Park & Pay is  $c = 0.5 + 0.25h$ . The equation for Greg's Garage is  $c = 1.1 + 0.1h$ .

5) a & b)



- The average height of a 16-year-old would be approximately 169 cm.
- The average height of a child  $11\frac{1}{2}$  years old is 145 cm.

### GEE 21 Connection

- d
- c
- c
- b
- d
- 5, 14, 5