

LOUISIANA MATHEMATICS FRAMEWORK

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SECTION I: INTRODUCTION

LOUISIANA CONTENT STANDARDS FOUNDATION SKILLS

The Louisiana Content Standards Task Force has developed the following foundation skills which should apply to all students in all disciplines.

1. **Communication**: A process by which information is exchanged and a concept of “meaning” is being created and shared between individuals through a common system of symbols, signs, or behavior. Students should be able to communicate clearly, fluently, strategically, technologically, critically, and creatively in society and in a variety of workplaces. This process can best be accomplished through use of the following skills: reading, writing, speaking, listening, viewing, and visually representing.
2. **Problem Solving**: The identification of an obstacle or challenge and the application of knowledge and thinking processes, which include reasoning, decision making, and inquiry in order to reach a solution using multiple pathways, even when no routine path is apparent.
3. **Resource Access and Utilization**: The process of identifying, locating, selecting, and using resource tools to help in analyzing, synthesizing, and communicating information. The identification and employment of appropriate tools, techniques, and technologies are essential to all learning processes. These resource tools include pen, pencil, and paper; audio/video material, word processors, computers, interactive devices, telecommunication, and other emerging technologies.
4. **Linking and Generating Knowledge**: The effective use of cognitive processes to generate and link knowledge across the disciplines and in a variety of contexts. In order to engage in the principles of continual improvement, students must be able to transfer and elaborate on these processes. “Transfer” refers to the ability to apply a strategy or content knowledge effectively in a setting or context other than that in which it was originally learned. “Elaboration” refers to monitoring, adjusting, and expanding strategies into other contexts.
5. **Citizenship**: The application of the understanding of the ideals, rights, and responsibilities of active participation in a democratic republic that includes working

respectfully and productively together for the benefit of the individual and the community; being accountable for one's choices and actions and understanding their impact on oneself and others; knowing one's civil, constitutional, and statutory rights; and mentoring others to become productive citizens and lifelong learners.

Note: These foundation skills are listed numerically in parentheses after each benchmark.

INFORMATION LITERACY MODEL FOR LIFELONG LEARNING

Students must become competent and independent users of information to be productive citizens of the 21st century. They must be prepared to live in an information-rich and changing global society. Due to the rapid growth of technology, the amount of information available is accelerating so quickly that teachers are no longer able to impart a complete knowledge base in a subject area. In addition, students entering the workforce must know how to access information, solve problems, make decisions, and work as part of a team. Therefore, information literacy -- the ability to recognize an information need and then locate, evaluate, and effectively use the needed information -- is a basic skill essential to the 21st century workplace and home. Information literate students are self-directed learners who, individually or collaboratively, use information responsibly to create quality products and to be productive citizens. Information literacy skills must not be taught in isolation; they must be integrated across all content areas, utilizing fully the resources of the classroom, the school library media center, and the community. The Information Literacy Model for Lifelong Learners is a framework that teachers at all levels can apply to help students become independent lifelong learners.

1. **Defining/Focusing:** The first task is to recognize that an information need exists. Students make preliminary decisions about the type of information needed based on prior knowledge.
2. **Selecting Tools and Resources:** After students decide what information is needed, they then develop search strategies for locating and accessing appropriate, relevant sources in the school library media center, community libraries and agencies, resource people, and others as appropriate.
3. **Extracting and Recording:** Students examine the resources for readability, currency, usefulness, and bias. This task involves skimming or listening for key words, “chunking” reading, finding main ideas, and taking notes.
4. **Processing Information:** After recording information, students must examine and evaluate the data in order to utilize the information retrieved. Students must interact with the information by categorizing, analyzing, evaluating, and comparing for bias, inadequacies, omissions, errors, and value judgments. Based on their findings, they either move on to the next step or do additional research.

5. **Organizing Information:** Students effectively sort, manipulate, and organize the information that was retrieved. They make decisions on how to use and communicate their findings.
6. **Presenting Findings:** Students apply and communicate what they have learned (e.g., research report, project, illustration, dramatization, portfolio, book, book report, map, oral/audio/visual presentation, game, bibliography, hyperstack).
7. **Evaluating Efforts:** Throughout the information problem solving process, students evaluate their efforts. This assists students in determining the effectiveness of the research process. The final product may be evaluated by the teacher and also other qualified or interested resource persons.

PHILOSOPHY

In mathematics classrooms in Louisiana, each student actively participates in a learning environment guided by a capable teacher and supported by the home and community. The student values mathematics and is confident and competent in his or her ability to use mathematics in an ever-changing world. The student develops mathematical understanding through individual and group instruction that includes investigating, discovering, communicating, and reasoning. Assessment, an integral part of the teaching and learning process, is carefully integrated with instructional practices.

NEED AND CONTEXT FOR REFORM

Rationale for Change

The rationale for change in mathematics education is driven by the implications of the evolving ages of technology and information and their implications for future societal and work force needs. “In tomorrow’s world, the best opportunities for jobs and advancement will go to those best prepared to cope confidently and competently with mathematical, scientific, and technological issues” (Everybody Counts, 1989). To adequately prepare students for the future, mathematics education must change to include the following:

- ! the student as an active participant in learning rather than a passive recipient of knowledge;**
- ! equitable access for all students to manipulatives and state-of-the-art technologies, including electronic networking; and**
- ! the incorporation of a variety of individual and group activities that use real-life experiences to develop critical thinking.**

Because the demands of the workplace are changing, our efforts to prepare students for the workplace must continually change. Schools must prepare students to adapt productively to change by focusing on the process of lifelong learning. The nation’s business leaders agree that to become productive workers and informed citizens in today’s society, students must develop the ability to:

- ! reason critically and understand concepts;**
- ! work with others;**
- ! communicate ideas effectively;**
- ! understand and interpret statistical information;**
- ! become lifelong learners; and**
- ! adapt to a dynamic work environment.**

Implications for Curricula Change

The processing of vast amounts of numerical information available through modern technology makes more imperative than ever the ability to synthesize mathematical information as a basis for rational decision making. Thus, quantitative thinking is becoming more pervasive in virtually all aspects of the workplace and everyday life experiences. School mathematics must remain attuned to the needs of students, adjusting to include the handling and understanding of data; the appreciation, recognition, and use of numerical and geometrical patterns; and the integration and synthesis of information leading to creative problem solving.

The basic facts of addition, subtraction, multiplication, and division are important; however, technology, specifically the development of calculators, allows all students to expand and extend much of traditional school mathematics far beyond the basic math facts and repeated drill and practice. Students should concentrate on understanding ideas, reasoning, solving problems, communicating, and making connections within mathematics and between mathematics and its growing applications in other fields.

Numbers and a sense of numbers are of greater value than ever before. Number sense -- the intuitive, meaningful use of numbers in mental computation, estimation, problem solving, and applications is vital. It is essential for students to develop this intuitive sense in order to determine, for example, if a number in a news account, on a printout, or on a display screen is appropriate and acceptable.

National Direction

In the 1980s after reports concerning the low performance of American students on international assessments, several publications emerged that directly addressed the urgent national need to revitalize mathematics education. The nation recognized that to be competitive in a global economy, American students had to be prepared to work competently and confidently in the age of technology and information. The most significant publications include the following:

- 1. Curriculum and Evaluation Standards for School Mathematics. (1989). National Council of Teachers of Mathematics.**
- 2. Everybody Counts: A Report to the Nation on the Future of Mathematics Education. (1989). National Research Council.**
- 3. Reshaping School Mathematics: A Philosophy and Framework for Curriculum. (1990). Mathematical Sciences Education Board.**

4. **Professional Standards for Teaching Mathematics.** (1991). National Council of Teachers of Mathematics.
5. **Mathematics Assessment.** (1991). National Council of Teachers of Mathematics.
6. **Assessment Standards for School Mathematics.** (1995). National Council of Teachers of Mathematics.

The Louisiana Mathematics Framework is based on the direction reflected in these and other reform-based publications.

Collaborative Systemic Reform

In 1990, the National Science Foundation (NSF) solicited proposals for Statewide Systemic Initiatives (SSI) Programs. In its program solicitation, the NSF described the proposals eligible for funding, stating that the initiatives “...must involve all those who have a responsibility to the system or to particular parts of it whether at the state or local level.” It further stated that partners involved in the initiatives must include “state leaders; teachers and other school system leaders; university faculty; leaders in science-rich institutions, including business and industry; and leaders of parent groups and other community based organizations.” The change to funding “systemic” efforts to involve all major stakeholders in education was a major shift by NSF to effect reform of mathematics and science education throughout the nation.

The reform effort in Louisiana began with a successful application to establish an NSF-funded statewide systemic initiative. A broad-based coalition of Louisianians worked to secure a five-year, \$10 million grant for the Louisiana Systemic Initiatives Program (LaSIP). Louisiana provided matching funds from the Louisiana Board of Elementary and Secondary Education (LBESE) and the Louisiana Board of Regents (LBoR) for colleges and universities to support the reform of mathematics and science education.

The Louisiana Department of Education (LDE) was awarded a Dwight D. Eisenhower National Program for Mathematics and Science Education grant from the United States Department of Education. This grant enabled the LDE, in collaboration with LaSIP, to develop the Louisiana Mathematics and Science Curriculum and Assessment Frameworks.

Framework Development Process

Using recommendations from statewide leaders in mathematics, panel members were selected to assist in the development of the Louisiana Mathematics and Science Frameworks. The panel was divided into subpanels for mathematics and science, consisting of approximately 40 members each. Membership consisted of: LDE personnel; LaSIP staff; educators from both public and private schools; educators who had expertise in working with students with disabilities; educators with expertise in working with minorities and underserved populations; and university faculty from mathematics, the sciences, and education.

After assessing the current state of affairs in Louisiana in light of national reform, the mathematics subpanel began the development of a strategic plan to reform mathematics curricula in Louisiana. Two 15-member Mathematics Framework Steering Committees, consisting of LDE staff, LaSIP staff, university faculty, supervisors, and classroom teachers, were formed to oversee the writing of the Framework and grade-level handbooks. As drafts of the Framework were completed, extensive reviews were conducted by state educational stakeholders and national leaders in mathematics reform. In collaboration with the steering committee, exemplary classroom teachers helped write grade-level handbooks. Drafts of the handbooks were reviewed by classroom teachers from across the state.

PURPOSE

This Framework document was formulated to articulate the shared vision of the mathematical, business, professional, and vocational communities of Louisiana concerning mathematics education. It provides a unifying structure which encompasses instructional methodologies and course content, while maintaining sufficient flexibility to permit adaptability within local districts. The Framework should guide the teacher in designing a comprehensive program that assists in the development of the mathematical power of each student. Using national mathematics standards as a guide, the Framework forms the foundation of a comprehensive mathematics educational program upon which state mathematics assessment can be based.

Intended Audience

The Louisiana Mathematics Framework is intended for a broad audience: teachers, curriculum supervisors, school and district administrators, school boards, business and industry leaders, parents, college, university, and state education agency staff and policy makers. With the Framework as a common reference point, it will be possible for these varied

groups to work to achieve a shared vision of what and how mathematics should be taught in Louisiana schools.

Intended Use

The Louisiana Mathematics Framework serves as a guide for curriculum and instruction and as a general reference to the basic principles of mathematics education.

Intended uses for this Framework include the following:

- ! **For teachers**, a guide for planning curriculum, instruction, and assessment;
- ! **For parents**, a means for assessing the effectiveness of their children’s mathematics education;
- ! **For administrators and school board members**, a vision for mathematics education and a basis for planning resource allocations, materials purchases, local curriculum development, and teachers’ professional development;
- ! **For policy makers and state education staffs**, a basis for developing laws, policies, and funding priorities to support local reforms;
- ! **For staff developers**, a basis for creating professional development materials and strategies designed to increase teachers’ knowledge of mathematics content, teaching methodologies and assessment strategies;
- ! **For assessment specialists and test developers**, a guide to establishing tools and strategies that effectively assess students’ mathematical understanding and ability;
- ! **For colleges and universities**, a guide for content and design of teacher preparation programs; and
- ! **For business and industry leaders and governmental agencies**, a basis for developing effective partnerships and local reforms for funding instructional materials and professional development.

FRAMEWORK CRITERIA

The following criteria, which provided the foundation for the development of the Framework, are critical to strengthen, support, and sustain mathematics education.

- ! The Louisiana Mathematics Framework reflects national standards in defining K-12 curricula.**
- ! A National Validation Team consisting of nationally recognized mathematics educators and mathematicians reviewed the documents during development to ensure content validity.**
- ! The Louisiana Mathematics Framework is equitable for all students.**

In addition to involving representatives from under represented groups in the development process, a Louisiana Equity Review Team, consisting of state leaders representing the following groups, reviewed the documents: students with learning disabilities, students with special education needs (including disabled and gifted students), minorities, students who speak English as a second language, and women. Professional development activities for Framework implementation will include specific strategies to assist the teacher in addressing the needs of all students.

- ! Classroom teachers are significantly involved in the development of the Louisiana Mathematics Framework.**

Through the organizational structures of the Louisiana Department of Education (LDE) and the Louisiana Systemic Initiatives Program (LaSIP), the drafts of the documents were reviewed by over 2,000 mathematics teachers from throughout the state.

- ! The Louisiana Mathematics Framework includes a comprehensive, well-developed structure that demonstrates cohesiveness and continuity from kindergarten through 12th grade.**

University faculty were an integral part of the framework development process. Classroom teachers from grades K-4, 5-8, and 9-12 were involved in the development of all components of the framework.

- ! **The Louisiana Mathematics Framework demonstrates the relevance of mathematics to real-life activities.**

The Framework and handbooks have a pervasive theme of interconnectedness to real-life situations. The classroom activities included in the handbooks engage students in mathematical activities that are relevant and genuinely motivating. The documents were reviewed by representatives from Louisiana business and industry to ensure their relevance to activities in which specific mathematical principles are applied.

- ! **The Louisiana Mathematics Framework reflects national trends in assessment by thoroughly integrating assessment and instruction.**

The handbooks include grade-level alternative assessment samples for both classroom and large-scale assessment. Several nationally recognized leaders in student assessment reviewed the documents.

- ! **The Louisiana Mathematics Framework is dynamic and easily adaptable to future changes that better prepare both teachers and students to be lifelong learners.**

Representatives from the LDE and the Louisiana Association of Teachers of Mathematics (LATM) will convene each year, as needed at the annual LATM meeting to review the Framework to ensure that it remains dynamic. Revisions will be transmitted electronically to each district.

PERVASIVE THEMES

The vision of mathematics education in this Framework is expressed through five pervasive and thoroughly interwoven themes which encompass the strands of school mathematics.

- ! **Mathematics as Problem Solving**

Classroom instruction should focus on more diverse and complex problem-solving situations that arise from relevant, real-life circumstances. Students should be able to design problems and generate appropriate solutions. For a given problem, teachers must actively encourage students to find alternative approaches to the problem, as well as using formal procedures.

! **Mathematics as Numerical Intuition**

Students should develop a common-sense approach to using numbers, an intuitive feel for numbers including various uses and meanings, an appreciation for different levels of accuracy needed, and the ability to determine the reasonableness of answers.

! **Mathematics as Reasoning**

Students should use critical thinking skills in questioning, elaborating, validating, and justifying.

! **Mathematics as Connections**

Topics within mathematics should be interconnected rather than taught in isolation. Additionally, problems and procedures should be connected to other subject areas and to real-life, relevant situations that are challenging and motivating to the student.

! **Mathematics as Communication**

Students should be provided opportunities to express their mathematical ideas through speaking, writing, demonstrating, and modeling.

SECTION II: EDUCATIONAL ENVIRONMENT

VISION OF TEACHING

The teacher must have the content knowledge and skills to be an instructor and facilitator of mathematical learning. The teacher must have the necessary supplies and materials to encourage individual and group explorations by the students. The teacher allows time for students to investigate mathematical ideas or tasks and encourages the use of models, materials, and technology. The teacher ensures an environment that encourages risk-taking, questioning, discovery, and cooperation. The teacher listens and values all students' ideas and encourages students to construct understandings based on their personal learning style and prior experiences.

The teacher demonstrates the connectedness of mathematics by utilizing instructional activities that encompass benchmarks from several strands. These activities may require several days or weeks to complete, depending on grade-level appropriateness. While some exercises or independent problems will be used, they are embedded in large problems or issues that are relevant to the student.

VISION OF LEARNING

The classroom experience envisioned in this document is a dynamic one in which students become autonomous learners, while capable and empowered teachers guide them in taking charge of their own quest for knowledge. Students work independently, in small groups, or in large groups on problem-solving investigations. They have the materials and appropriate manipulatives to explore problems. They become risk takers through exploring ideas, forming questions, making and supporting conjectures, and learning to communicate and reason mathematically.

With problem solving at the heart of the curriculum, students develop an understanding of relevant problem-solving strategies including, but not limited to, the following: draw a picture or diagram; develop a chart, list, or table; guess and check; work backwards; simplify problems; use manipulatives, etc. Both student-generated and teacher-generated strategies are explored in developing an understanding of the various approaches to solving a problem.

EQUITY

Regardless of ability level, all students in Louisiana must be exposed to a challenging and motivating mathematics curriculum based on relevant problem-solving situations. Traditionally, high-achieving students have participated in activities that required critical thinking and reasoning, whereas students working at or below grade level spent large amounts of time in drill-and-practice activities. One of the myths identified in Mathematics Assessment, (NCTM, 1991) that abounds in mathematics education is that “problems and applications come only after mastery of skills.” Research supports the position that students learn skills and content in the context of challenging and motivating problems. The Louisiana Framework advocates a common core of significant mathematics that actively and interestingly engages all students.

In addition to having access to the common core curriculum, all students should have equal access to resources, qualified teachers, and quality instructions. The teacher is instrumental in creating an environment that encourages and facilitates each student’s mathematical development. Problem-solving situations should reflect and build upon real-life experiences of all students and should reflect diverse cultures.

The ability to learn mathematics is not determined by one’s socioeconomic level, gender, or ethnic origin. The teacher models the belief that all students can learn and demonstrates an appreciation and understanding of cultural diversity and varied learning styles. By challenging all students, the teacher creates the environment in which all students learn to approach mathematics with enthusiasm and confidence.

TECHNOLOGY

In past decades, the classroom environment was a reflection of the workplace. Employees at factories worked independently on routing assembly lines to construct products, while mathematics classrooms consisted of rows of students working independently on routine practice problems. The age of technology has dramatically changed the workplace environment. Employees now work cooperatively and use a variety of techniques to solve real, nonroutine problems. Classrooms must reflect these changes to prepare students for the 21st century.

The relevance of technology is expressed in the following underlying beliefs:

- ! Calculators and computers are basic tools of today’s mathematics just as paper, pencil, and slide rules were basic tools of past years.**
- ! Calculators and computers have reduced the need to make precise calculations by hand, but in doing so they have increased the importance of acquiring a well-developed number sense. (Goldsmith, 1992).**
- ! Appropriate calculators and computers should be available for all students.**
- ! Appropriate use of technology should be naturally integrated into the teaching of mathematics to assist the student to investigate and solve problems, not simply to check answers or to practice skills.**
- ! As resources become available via telecommunications, they should be used in the mathematics classroom to support standards-based instruction.**

SUPPORT STRUCTURES

The collaborative effort of all stakeholders is imperative if significant change is to occur in Louisiana schools. Support of mathematics reform must be demonstrated by teachers, students, administrators, school boards, parents, business and industry, elected officials, the media, community organizations, etc. The state and local communities must commit to long-range planning to schedule time for appropriate staff development, to ensure funds for necessary resources, and to provide appropriate learning environments and facilities for students. A professional development model, with an accompanying dissemination plan, has been designed by the LDE and LaSIP to support teachers as they implement the framework.

Louisiana is developing a state school improvement plan to address content, performance, and opportunity-to-learn standards in all subject disciplines. The Mathematics Framework will be a critical component of the state school improvement plan. As each local district develops its mathematics curriculum, the new curriculum should be aligned with the state Mathematics Framework and should become a part of the local school improvement plan.

SECTION III: ASSESSMENT

PURPOSE

Assessment in mathematics is a process through which evidence is gathered about a student's understanding and ability to apply that understanding. The changes in mathematics content and in the way mathematics is taught must be reflected through accompanying changes in assessment. Assessment is an ongoing, dynamic process which is both diagnostic and prescriptive in nature. It communicates, illustrates, and identifies the mathematics that is most important for students to learn and enhances mathematics learning. Assessment and instruction must be intertwined so that each supports the other in promoting the development of mathematical power for all students. Various assessment techniques should be used to:

- ! improve teaching and learning;**
- ! evaluate student progress;**
- ! assist in making decisions regarding individual student performance;**
- ! provide information on the effectiveness of educational programs;**
- ! provide data relative to the progress toward established educational goals;**
- ! address accountability issues; and**
- ! address the appreciation and understanding of various cultural differences and learning styles.**

CLASSROOM ASSESSMENT

Educational purposes for assessment may be as varied as assessment techniques. Assessment is the link between teaching and learning and provides information for making instructional decisions, monitoring student progress, and communicating student progress to appropriate audiences.

Assessment is moving away from the use of a single type of instrument to assess students' understanding and toward the use of a wide range of assessment techniques that require students to demonstrate critical thinking skills. Combinations of the following techniques,

integrated with instruction, can provide a comprehensive assessment of student understanding: observations, oral questions, journals, portfolios, multiple choice tests, projects, activities, concept maps, presentations, etc.

LARGE-SCALE ASSESSMENT

Large-scale assessment refers to assessment at the district, state, and national level. This type of assessment is used as an external monitoring of student progress on criteria established outside the classroom but with teacher input.

When external monitoring is aligned with the curriculum and teaching strategies, it has a positive impact on mathematics education. Appropriate use of external monitoring will enhance learning by providing external support for the teacher's own monitoring of student progress. The monitoring of student progress externally through established performance standards can provide the following:

- ! a measurement of student performance on a dynamic, authentic curriculum;**
- ! information for decision makers;**
- ! a measure of pupil progression;**
- ! a criterion for graduation;**
- ! information for education program evaluation;**
- ! demographic data;**
- ! system-wide data; and**
- ! data for national comparisons.**

SECTION IV: CONTENT STRANDS

The six content strands translate the vision of the new mathematics curriculum: Number and Number Relations (N); Algebra (A); Measurement (M); Geometry (G); Data Analysis, Probability, and Discrete Math (D); and Patterns, Relations, and Functions (P). Each of the strands is introduced with a focus statement followed by the standard and benchmarks for that strand.

The strands are intended to be thoroughly interwoven, providing rich connections at all grade levels. There should be deliberate reinforcement of concepts throughout the school year.

Although the content is delineated by strands, it is not a recipe to be followed line by line. Instead, the content provides the building blocks upon which a dynamic, cohesive, and comprehensive mathematics program can be built. It supports student explorations and investigations that relate objectives from several strands. The very nature of the content implies that concepts and understandings should not be taught in isolation.

To assist teachers, a handbook that contains sample classroom activities has been developed. The handbook will assist the teacher to translate the content into standards-based classroom instruction. Three or four activities are included for each grade level. The Framework and sample activities should assist the teacher in developing curriculum and instruction that enhance the mathematical power of all students.

NUMBER AND NUMBER RELATIONS

FOCUS

Developing an intuitive, common-sense approach to number relationships and operations is of primary importance and should permeate every area of the mathematics curriculum. Number sense involves the use of “friendly easy numbers” and of actively seeking alternative ways of making computations. Number sense is not a topic to be taught as a unit, but is a prevailing theme throughout all mathematics. All students should develop a conceptual understanding of number magnitude and number operations through participation in hands-on investigative activities. These activities should provide many opportunities for students to discover and develop problem-solving strategies. Student involvement in these activities should assist in the development of estimation skills (particularly when an approximate answer is sufficient) and other mental arithmetic skills (when an exact answer is required). When the numbers are not manageable for mental arithmetic and an exact answer is required, calculators or paper and pencil should be used. Parallel with the need to develop an understanding of the methods and usage of various computational techniques is the students’ need for an informal development of mathematical language and symbolism. Inherent in our increasing dependence on technology is the danger of accepting machine answers at face value. A well-developed number sense can combat this danger. Furthermore, number sense leads naturally to the development of symbol sense necessary for use with technology, such as graphing calculators and symbolic manipulators. This developing mathematical power will allow the students to function and communicate more effectively and with greater confidence in real-life experiences.

STANDARD

In problem-solving investigations, students demonstrate an understanding of the real number system and communicate the relationships within that system using a variety of techniques and tools.

BENCHMARKS K-4

Students in Grades K-4 use estimation, mental arithmetic, number lines, graphs, appropriate models, manipulatives, calculators, and computers as they investigate problems involving whole numbers. As a result, what they know and are able to do includes:

- N-1-E** constructing number meaning and demonstrating that a number can be expressed in many different forms (e.g., standard notation, number words, number lines, geometrical representation, fractions, and decimals);
(1, 2, 4)
- N-2-E** demonstrating number sense and estimation skills, giving particular attention to common equivalent reference points (i.e., $1/4 = 25\% = .25$; $1/2 = 50\% = .5$; $\$1 = 100\%$, etc.);
(1)
- N-3-E** reading, writing, representing, comparing, ordering, and using whole numbers in a variety of forms (e.g., standard notation, number line, and geometrical representation);
(1, 4)
- N-4-E** demonstrating a conceptual understanding of the meaning of the basic arithmetic operations (add, subtract, multiply, and divide) and their relationships to each other;
(1)
- N-5-E** selecting appropriate operation(s) (add, subtract, multiply, and divide) for a given situation;
(2, 3, 4)
- N-6-E** applying a knowledge of basic math facts and arithmetic operations to real-life situations;
(2, 4, 5)
- N-7-E** constructing, using, and explaining procedures to compute and estimate with whole numbers (e.g., mental math strategies)
(1, 4)
- N-8-E** selecting and using appropriate computational methods and tools for given situations involving whole numbers (e.g., estimation, mental arithmetic, calculator, or paper and pencil);
(2, 4)
- N-9-E** demonstrating the connection of number and number relations to the other strands and to real-life situations.

(1, 4, 5)

BENCHMARKS 5-8

Students in Grades 5-8 use estimation, mental arithmetic, number lines, graphs, appropriate models, manipulatives, calculators, and computers as they extend their investigations of problems involving rational numbers. As a result, what they know and are able to do includes:

- N-1-M demonstrating that a rational number can be expressed in many forms, and selecting an appropriate form for a given situation (e.g., fractions, decimals, and percents);
(1, 2, 4)**
- N-2-M demonstrating number sense and estimation skills to describe, order, and compare rational numbers (e.g., magnitude, integers, fractions, decimals, and percents);
(2, 4)**
- N-3-M reading, writing, representing, and using rational numbers in a variety of forms (e.g., integers, mixed numbers, and improper fractions);
(1)**
- N-4-M demonstrating a conceptual understanding of the meaning of the basic arithmetic operations (add, subtract, multiply and divide) and their relationships to each other;
(1, 2)**
- N-5-M applying an understanding of rational numbers and arithmetic operations to real-life situations;
(1, 2, 3, 4)**
- N-6-M constructing, using, and explaining procedures to compute and estimate with rational numbers employing mental math strategies;
(1, 2, 3, 4)**
- N-7-M selecting and using appropriate computational methods and tools for given situations involving rational numbers (e.g., estimation, or exact computation using mental arithmetic, calculator, computer, or paper and pencil);
(2, 3, 4)**

N-8-M demonstrating a conceptual understanding and applications of proportional reasoning (e.g., determining equivalent ratios, finding a missing term of a given proportion).
(2, 4)

BENCHMARKS 9-12

Students in Grades 9-12 use estimation, mental arithmetic, number lines, graphs, appropriate models, manipulatives, calculators, and computers as they extend their investigations of problems involving real numbers. As a result, what they know and are able to do includes:

N-1-H demonstrating an understanding of the real number system;
(1, 2, 4)

N-2-H demonstrating that a number can be expressed in many forms, and selecting an appropriate form for a given situation (e.g., fractions, decimals, percents, and scientific notation);
(1, 4)

N-3-H using number sense to estimate and determine if solutions are reasonable;
(2, 4)

N-4-H determining whether an exact or approximate answer is necessary;
(2, 3, 4)

N-5-H selecting and using appropriate computational methods and tools for given situations (e.g., estimation, or exact computation using mental arithmetic, calculator, symbolic manipulator, or paper and pencil);
(3)

N-6-H applying ratios and proportional thinking in a variety of situations (e.g., finding a missing term of a proportion);
(2, 4)

N-7-H justifying reasonableness of solutions and verifying results.
(1, 2, 4)

ALGEBRA

FOCUS

Algebra is much more than the study of generalized forms of arithmetic. It is a powerful language used to interpret real-world experience. This language is a communication tool used to analyze and describe relationships and mathematical structures. Beginning at the elementary level, the school mathematics curriculum should integrate the use of the language of Algebra throughout all strands of the curriculum to enable students to shift progressively from informal to formal concepts and from concrete to symbolic representations. The middle school mathematics curriculum should integrate the use of this language throughout all strands of the curriculum to enable students to progressively shift from the concrete to the symbolic. At this level, algebra should be conceptual and intuitive, not formally computational. It should involve actively seeking easy and alternative ways of looking at problems. These transitions should be powered by investigations involving the use of appropriate manipulatives, models, and technology, and should encourage the development of communication, reasoning, and problem-solving skills. Algebra, in the K-8 classrooms, refers to informal explorations and understandings of symbolism. It is beneficial to introduce the algebraic terminology (equation, inequality, variable, etc.) in the early grades. In this way high school students will be able to understand algebra as a natural outgrowth of their study of various number properties. The high school curriculum should continue the development of symbolic representatives. The use of modern technology frees teachers and students from the need to develop complicated pencil and paper manipulative skills in algebra. More classroom time is now available to apply algebra in solving challenging real-world problems. This will allow students to recognize the worth, importance, and power of the mathematics of abstraction and symbolism.

STANDARD

In problem-solving investigations students demonstrate an understanding of concepts and processes that allow them to analyze, represent, and describe relationships among variable quantities and to apply algebraic methods to real-world situations.

BENCHMARKS K-4

Students in Grades K-4 use manipulatives, models, graphs, tables, technology, number sense, and estimation as they investigate problems involving the concepts and application of algebra. As a result, what they know and are able to do includes:

- A-1-E** demonstrating a conceptual understanding of variables, expressions, equations, and inequalities (e.g., use letters or boxes to represent values; understand =, ..., <, and > symbols);
(1, 4)
- A-2-E** modeling and developing strategies for solving equations and inequalities;
(1, 2, 3, 4)
- A-3-E** recognizing the connection of algebra to the other strands and to real-life situations (e.g., number sentences or formulas to represent real-world problems).
(4, 5)

BENCHMARKS 5-8

Students in Grades 5-8 use manipulatives, models, graphs, tables, technology, number sense, and estimation as they extend their investigations of problems involving the concepts and application of algebra. As a result, what they know and are able to do includes:

- A-1-M** demonstrating a conceptual understanding of variables, expressions, equations, and inequalities (e.g., symbolically represent real-world problems as linear terms, equations, or inequalities);
(1, 2, 4)
- A-2-M** modeling and developing methods for solving equations and inequalities (e.g., using charts, graphs, manipulatives, and/or standard algebraic procedures);
(2, 3, 4)
- A-3-M** representing situations and number patterns with tables, graphs, and verbal and written statements, while exploring the relationships among these representations (e.g., multiple representations for the same situation);
(1,4)
- A-4-M** analyzing tables and graphs to identify relationships exhibited by the data and making generalizations based upon these relationships;
(2, 3, 4)
- A-5-M** demonstrating the connection of algebra to the other strands and to real-life situations.

(1, 2, 3, 4, 5)

BENCHMARKS 9-12

Students in Grades 9-12 use manipulatives, models, graphs, tables, technology, number sense, and estimation as they extend their investigations of problems involving the concepts and application of algebra. As a result, what they know and are able to do includes:

- A-1-H demonstrating the ability to translate real-world situations (e.g., distance versus time relationships, population growth, growth functions for diseases, growth of minimum wage, auto insurance tables) into algebraic expressions, equations, and inequalities and vice versa;
(1, 2, 4)**
- A-2-H recognizing the relationship between operations involving real numbers and operations involving algebraic expressions;
(2, 4)**
- A-3-H using tables and graphs as tools to interpret algebraic expressions, equations, and inequalities;
(1, 3)**
- A-4-H solving algebraic equations and inequalities using a variety of techniques with the appropriate tools (e.g., hand-held manipulatives, graphing calculator, symbolic manipulator, or pencil and paper).
(2, 3)**

MEASUREMENT

FOCUS

Measurement is the connection between numbers and the real world and as such is a vital component of an attempt to organize the world. It allows one to communicate effectively and make decisions. It relates geometry and algebra, as well as geometry and numbers, in both intuitive and formal ways. It is also a connecting theme between such diverse fields as athletics, music, travel, astronomy, and engineering. The study of measurement should consist of active investigations based on real-world problems in both individual and group format. These explorations should include the appropriate use of manipulatives and technology and should encourage the development of communications, reasoning, and problem-solving skills. Students need to learn the effect of unit choice on mathematical entities, such as the shape of graphs and the magnitude of answers. Secondary students should become so adept with the use of units that they are comfortable with the use of compound units (foot-pounds, miles per second) and specialized units (atmospheres, millennia, gigabytes) as they occur in real-world problems.

STANDARD

In problem-solving investigations, students demonstrate an understanding of the concepts, processes, and real-life applications of measurement.

BENCHMARKS K-4

Students in Grades K-4 use number sense, estimation, appropriate manipulatives, tools, and technology as they investigate problems involving measurement. As a result, what they know and are able to do includes:

- M-1-E** applying (measure or solve measurement problem) the concepts of length (inches, feet, yards, miles, millimeters, centimeters, decimeters, meters, kilometers), area, volume, capacity (cups, liquid pints and quarts, gallons, milliliters, liters), weight (ounces, pounds, tons, grams, kilograms), mass, time (seconds, minutes, hours, days, weeks, months, years), money, and temperature (Celsius and Fahrenheit) to real-world experiences;
(1, 2, 3, 4, 5)

- M-2-E** selecting and using appropriate standard and non-standard units of measure (e.g., paper clips and Cuisenaire rods) and tools for measuring length, area, capacity, weight/mass, and time for a given situation by considering the purpose and precision required for the task;
(1, 2, 3, 4)
- M-3-E** using estimation skills to describe, order, and compare measures of length, capacity, weight/mass, time, and temperature;
(1, 2, 3, 4)
- M-4-E** converting from one unit of measurement to another within the same system (customary and metric); comparisons between systems should be based on intuitive reference points, not formal computations (e.g., a meter is a little longer than a yard);
(2, 3, 4)
- M-5-E** demonstrating the connection of measurement to the other strands and to real-life situations.
(2, 4, 5)

BENCHMARKS 5-8

Students in Grades 5-8 use number sense, estimation, appropriate manipulatives, tools, and technology as they extend their investigations of problems involving measurement. As a result, what they know and are able to do includes:

- M-1-M** applying the concepts of length, area, surface area, volume, capacity, weight, mass, money, time, temperature, and rate to real-world experiences;
(2, 3, 4)
- M-2-M** demonstrating an intuitive sense of measurement (e.g., estimating and determining reasonableness of measures);
(1, 2, 4)
- M-3-M** selecting appropriate units and tools for tasks by considering the purpose for the measurement and the precision required for the task (e.g., length of a room in feet rather than inches);
(2, 3, 4)

- M-4-M** using intuition and estimation skills to describe, order, and compare formal and informal measures (e.g., ordering cup, pint, quart, gallon; comparing a meter to a yard);
(1, 2, 4)
- M-5-M** converting from one unit of measurement to another within the same system (Comparisons between systems, customary and metric, should be based on intuitive reference points, not formal computation.);
(2, 4)
- M-6-M** demonstrating the connection of measurement to the other strands and to real-life situations.
(1, 2, 3, 4, 5)

BENCHMARKS 9-12

Students in Grades 9-12 use number sense, estimation, appropriate manipulatives, tools, and technology as they extend their investigations of problems involving measurement. As a result, what they know and are able to do includes:

- M-1-H** selecting and using appropriate units, techniques, and tools to measure quantities in order to achieve specified degrees of precision, accuracy, and error (or tolerance) of measurements;
(3)
- M-2-H** demonstrating an intuitive sense of measurement (e.g., estimating and determining reasonableness of results as related to area, volume, mass, rate, and distance);
(1, 2, 4)
- M-3-H** estimating, computing, and applying physical measurement using suitable units (e.g., calculate perimeter and area of plane figures, surface area and volume of solids presented in real-world situations);
(1, 3, 4)
- M-4-H** demonstrating the concept of measurement as it applies to real-world experiences.
(1, 2, 3, 4, 5)

GEOMETRY

FOCUS

Geometry is the study of the physical shapes of the world in which we live. It provides a natural environment for the use of inductive and deductive reasoning. It is not only basic to design, construction, and engineering, but also to law, medicine, and other fields that depend on critical deductive thinking skills. It provides models for representing many numerical and algebraic concepts. In Grades K-4, students must have opportunities to examine, manipulate, and construct geometric models using concrete materials. These activities should take place in a setting where students may freely explore and discuss ideas in order to develop and use appropriate vocabulary. After such first-hand experiences, many students should be able to progress to pictorial and abstract representations. The study of geometry should center around cooperative group investigations designed to promote the discovery of geometric concepts and principles and should encourage the development of communication, reasoning, and problem-solving skills. Secondary students should develop coordinate and transformational geometry as well as the usual axiomatic geometry. They should develop deductive reasoning skills by way of written proofs in a variety of formats. In the study of geometry, students should have access to appropriate manipulatives, technology, and construction materials to enhance their investigations.

STANDARD

In problem-solving investigations, students demonstrate an understanding of geometric concepts and applications involving one-, two-, and three-dimensional geometry, and justify their findings.

BENCHMARKS K-4

Students in Grades K-4 use number sense, estimation, models, drawings, manipulatives, and technology as they investigate problems involving geometric concepts. As a result, what they know and are able to do includes:

G-1-E determining the relationships among shapes;
(1, 2, 3, 4)

- G-2-E** identifying, describing, comparing, constructing, and classifying two-dimensional and three-dimensional geometric shapes using a variety of materials;
(1, 2, 3, 4)
- G-3-E** making predictions regarding combinations, subdivisions, and transformations (slides, flips, turns) of simple plane geometric shapes;
(1, 2, 4)
- G-4-E** drawing, constructing models, and comparing geometric shapes, with special attention to developing spatial sense;
(1, 2, 4)
- G-5-E** identifying and drawing lines and angles and describing their relationships to each other and to the real world;
(1, 4, 5)
- G-6-E** demonstrating the connection of geometry to the other strands and to real-life situations.
(1, 2, 3, 4, 5)

BENCHMARKS 5-8

Students in Grades 5-8 use number sense, estimation, models, drawings, manipulatives, and technology as they extend their investigations of problems involving geometric concepts. As a result, what they know and are able to do includes:

- G-1-M** using estimation skills to describe, order, and compare geometric measures;
(1, 2)
- G-2-M** identifying, describing, comparing, constructing, and classifying geometric figures and concepts;
(1, 2, 3)
- G-3-M** making predictions regarding transformations of geometric figures (e.g., make predictions regarding translations, reflections, and rotations of common figures);
(1, 4)

- G-4-M** constructing two- and three-dimensional models;
(3)
- G-5-M** making and testing conjectures about geometric shapes and their properties;
(1, 2, 3, 4)
- G-6-M** demonstrating an understanding of the coordinate system (e.g., locate points, identify coordinates, and graph points in a coordinate plane to represent real-world situations);
(1, 3, 4)
- G-7-M** demonstrating the connection of geometry to the other strands and to real-life situations (e.g., applications of the Pythagorean Theorem).
(1, 3, 4, 5)

BENCHMARKS 9-12

Students in Grades 9-12 use number sense, estimation, models, drawings, manipulatives, and technology as they extend their investigations of problems involving geometric concepts. As a result, what they know and are able to do includes:

- G-1-H** identifying, describing, comparing, constructing, and classifying geometric figures in two and three dimensions using technology where appropriate to explore and make conjectures about geometric concepts and figures;
(1, 2, 3, 4)
- G-2-H** representing and solving problems using geometric models and the properties of those models (e.g., Pythagorean Theorem or formulas involving radius, diameter, and circumference);
(1, 2, 3)
- G-3-H** solving problems using coordinate methods, as well as synthetic and transformational methods (e.g., transform on a coordinate plane a design found in real-life situations);
(2)

- G-4-H** using inductive reasoning to predict, discover, and apply geometric properties and relationships (e.g., patty paper constructions, sum of the angles in a polygon);
(1, 2, 4)
- G-5-H** classifying figures in terms of congruence and similarity and applying these relationships;
(4)
- G-6-H** demonstrating deductive reasoning and mathematical justification (e.g., oral explanation, informal proof, and paragraph proof).
(1, 2, 4)

DATA ANALYSIS, PROBABILITY, AND DISCRETE MATH

FOCUS

Data analysis is the collecting, organizing, presenting, and analyzing of numerical information using appropriate statistical methods. Discrete mathematics is that branch of mathematics that involves finite sets and structured sets, including matrices and graph theory. Probability is that branch of mathematics that deals with uncertainty and the likelihood of events occurring or not occurring. These three subjects are closely interwoven. Concepts from these subjects should develop gradually through many varied experiences based on students' natural interests. These concepts are essential to help students relate mathematical thinking to real-life situations, such as weather, games, sports, newspapers, and business. Classroom explorations involving these concepts should encourage the development of communication, connections, reasoning, and problem-solving skills and should effectively incorporate the use of appropriate models, manipulatives, and technology. Talking and writing should be of particular importance in this strand as students learn to analyze information and express similarities, differences, and patterns based on their investigations. The concepts studies will enable students to effectively communicate information in an organized and graphic manner that will enhance problem-solving skills.

STANDARD

In problem-solving investigations, students discover trends, formulate conjectures regarding cause-and-effect relationships, and demonstrate critical thinking skills in order to make informed decisions.

BENCHMARKS K-4

Students in Grades K-4 use collection and organizational techniques, number sense, estimation, manipulatives, and technology as they investigate problems involving data. As a result, what they know and are able to do includes:

**D-1-E collecting, organizing, and describing data based on real-life situations;
(1, 3, 4, 5)**

**D-2-E constructing, reading, and interpreting data in charts, graphs, tables, etc;
(1, 2, 3, 4)**

- D-3-E** formulating and solving problems that involve the use of data;
(2, 3, 4)
- D-4-E** exploring, formulating, and solving sequence-of-pattern problems involving selection and arrangement of objects/numerals;
(2, 3, 4)
- D-5-E** predicting outcomes based on probability (e.g., make predictions of same chance, more likely, or less likely; determine fair and unfair games);
(1, 2, 4)
- D-6-E** demonstrating the connection of data analysis, probability, and discrete math to other strands and real-life situations.
(1, 2, 3, 4, 5)

BENCHMARKS 5-8

Students in Grades 5-8 use collection and organizational techniques, number sense, estimation, manipulatives, and technology as they extend their investigations of problems involving data. As a result, what they know and are able to do includes:

- D-1-M** systematically collecting, organizing, describing, and displaying data in charts, tables, plots, graphs, and/or spreadsheets;
(1, 2, 3, 4)
- D-2-M** analyzing, interpreting, evaluating, drawing inferences, and making estimations, predictions, decisions, and convincing arguments based on organized data (e.g., analyze data using concepts of mean, median, mode, range, random samples, sample size, bias, and data extremes);
(1, 2, 3, 4, 5)
- D-3-M** describing informal thinking procedures (e.g., solving elementary logic problems using Venn diagrams, tables, charts, and/or elementary logic operatives to solve logic problems in real-life situations; reach valid conclusions in elementary logic problems involving “and, or, not, if/then”);
(2, 3)

- D-4-M** analyzing various counting and enumeration procedures with and without replacement (e.g., find the total number of possible outcomes or possible choices in a given situation);
(2, 4)
- D-5-M** comparing experimental probability results with theoretical probability (e.g., representing probabilities of concrete situations as common fractions, investigating single-event and multiple-event probability, using sample spaces, geometric figures, tables, and/or graphs);
(2, 3, 4)
- D-6-M** demonstrating the connection of data analysis, probability, and discrete math to other strands and to real-life situations.
(1, 2, 3, 4, 5)

BENCHMARKS 9-12

Students in Grades 9-12 use collection and organizational techniques, number sense, estimation, manipulatives, and technology as they extend their investigations of problems involving data. As a result, what they know and are able to do includes:

- D-1-H** designing and conducting statistical experiments that involve the collection, representation, and analysis of data in various forms (Analysis should reflect an understanding of factors such as: sampling, bias, accuracy, and reasonableness of data.);
(1, 2, 3, 4)
- D-2-H** recognizing data that relate two variables as linear, exponential, or otherwise in nature (e.g., match a data set, linear or non-linear, to a graph and vice versa);
(1, 2, 3, 4)
- D-3-H** using simulations to estimate probabilities (e.g., lists and tree diagrams);
(1, 2, 3, 4)
- D-4-H** demonstrating an understanding of the calculation of finite probabilities using permutations, combinations, sample spaces, and geometric figures;
(1, 3)

- D-5-H** recognizing events as dependent or independent in nature and demonstrating techniques for computing multiple-event probabilities;
(1, 2, 4)
- D-6-H** recognizing and answering questions about data that are normally or non-normally distributed;
(1,2, 4)
- D-7-H** making inferences from data that are organized in charts, tables, and graphs (e.g., pictograph; bar, line, or circle graph; stem-and-leaf plot or scatter plot);
(1, 3, 4)
- D-8-H** using logical thinking procedures, such as flow charts, Venn diagrams, and truth tables;
(2, 3, 4)
- D-9-H** using discrete math to model real-life situations (e.g., fair games or elections, map coloring).
(1, 2, 3, 4, 5)

PATTERNS, RELATIONS, AND FUNCTIONS

FOCUS

The concepts of patterns, relations, and functions play a central role in modern mathematics. These concepts arise naturally from observations of the world. Business people, social scientists, and physical scientists use mathematics to make predictions following their study of patterns and relationships found among the quantities measured in their respective fields. In Grades K-8, students should use informal investigations to observe patterns created by nature and man (flowers, leaves, insects, music, predictable literature, wallpaper, fabric). Students should continue to use the study of patterns to explore mathematical relationships as they verbalize, complete, create, and analyze patterns. This gradual transition from the concrete to the symbolic provides a foundation for the study of functions. Not only does the high school curriculum contain the formal study of functions and inverse relations, it also uses functions and inverse relations as modeling tools for the study of relationships found in our world. This study of functions and how things change leads naturally to powerful analytic techniques, which are collectively called calculus.

STANDARD

In problem-solving investigations, students demonstrate an understanding of patterns, relations, and functions that represent and explain real-world situations.

BENCHMARKS K-4

Students in Grades K-4 use number sense, estimation, manipulatives, drawings, tables, graphs, formulas, and technology as they investigate problems involving patterns, relations, and functions. As a result, what they know and are able to do includes:

- P-1-E** recognizing, describing, extending, and creating a wide variety of numerical (e.g., skip counting of whole numbers), geometrical, and statistical patterns;
(1, 2, 3, 4)

- P-2-E** representing and describing mathematical relationships using tables, variables, open sentences, and graphs;
(1, 2, 4)

- P-3-E** recognizing the use of patterns, relations, and functions in other strands and in real-life situations.
(2, 3, 4, 5)

BENCHMARKS 5-8

Students in Grades 5-8 use number sense, estimation, manipulatives, drawings, tables, graphs, formulas, and technology as they extend their investigations of problems involving patterns, relations, and functions. As a result, what they know and are able to do includes:

- P-1-M** describing, extending, analyzing, and creating a wide variety of numerical, geometrical, and statistical patterns (e.g., skip counting of rational numbers and simple exponential number patterns);
(1, 2, 3, 4)
- P-2-M** describing and representing relationships using tables, rules, simple equations, and graphs;
(1, 3, 4)
- P-3-M** analyzing relationships to explain how a change in one quantity results in a change in another (e.g., change in the dimensions of a rectangular solid affects the volume);
(1, 2, 4)
- P-4-M** demonstrating the pervasive use of patterns, relations, and functions in other strands and in real-life situations.
(1, 4, 5)

BENCHMARKS 9-12

Students in Grades 9-12 use number sense, estimation, manipulatives, drawings, tables, graphs, formulas, and technology as they extend their investigations of problems involving patterns, relations, and functions. As a result, what they know and are able to do includes:

- P-1-H** modeling the concepts of variables, functions, and relations as they occur in the real world and using the appropriate notation and terminology;
(1, 3, 4)

- P-2-H** translating between tabular, symbolic, or graphic representations of functions;
(1, 3, 4)
- P-3-H** recognizing behavior of families of elementary functions, such as polynomial, trigonometric, and exponential functions, and, where appropriate, using graphing technologies to represent them;
(3, 4)
- P-4-H** analyzing the effects of changes in parameters (e.g., coefficients and constants) on the graphs of functions, using technology whenever possible;
(2, 3)
- P-5-H** analyzing real-world relationships that can be modeled by elementary functions.
(1, 3, 4)

SECTION V: GLOSSARY

<u>accuracy</u> (see precision)	Accuracy refers to relative error, that is, the maximum allowable error (tolerance) of a measurement divided by the measurement. (<i>For example, if a stick is measured to the nearest centimeter as 25 centimeters long, the accuracy of that measurement is one-half—the maximum error is half a centimeter—divided by 25, or two percent.</i>)
<u>algebra</u>	The branch of mathematics that is the symbolic generalization of the ideas of arithmetic.
<u>basic facts</u>	Addition facts through 10 ($0 + 0, 1 + 0, \dots, 10 + 10$), subtraction facts which are the inverses of the addition facts ($20 - 10, \dots, 1 - 0, 0 - 0$), multiplication facts ($1 \times 1, 1 \times 2, \dots, 10 \times 10$), and division facts which are the inverses of the multiplication facts ($1 \div 1, 2 \div 1, \dots, 100 \div 10$).
<u>coordinate geometry</u>	Geometry based on the coordinate system.
<u>data analysis</u>	The collection, organization, and interpretation of numerical data arising in the real world.
<u>discrete math</u>	The branch of mathematics dealing with countable sets including matrices, graph theory, and counting procedures.
<u>experimental probability</u>	Probability determined by collecting data from repeated trials of an experiment.
<u>function</u>	A relationship between two sets of numbers (or other mathematical objects). Functions can be used to understand how one quantity varies in relation to another (<i>for example, the relationship between the number of cars and the number of tires</i>). Once a member of the first set is chosen, the associated member of the second set is uniquely determined.
<u>integers</u>	The set of numbers consisting of the counting numbers (that is, 1, 2, 3, 4, 5, ...), their opposites (that is, negative numbers, -1, -2, -3, ...), and zero.

<u>intuitive</u>	Perceived insight or awareness.
<u>magnitude</u>	Size or largeness.
<u>measurement</u>	A way of quantifying the world in which we live.
<u>patty paper</u>	Thin, waxy squares of paper used in geometric constructions (e.g., hamburger paper).
<u>precision</u> (see accuracy)	The precision of a measurement is determined by the size of the unit used. The smaller the unit, the more precise the measurement. Precision refers to the fineness of the measurement and is limited by the measuring instrument used.
<u>rational number</u>	A number that can be expressed in the form a/b , where a and b are integers and $b \neq 0$ (for example, $3/4$, $2/1$, or $11/3$). Every integer is a rational number, since it can be expressed in the form a/b (for example, $5 = 5/1$). Rational numbers may be expressed as fractional or decimal numbers (for example, $3/4$ or $.75$). Finite decimals, repeating decimals, and mixed numbers all represent rational numbers.
<u>reflection</u> (also called a flip)	A transformation which produces the mirror image of a geometric figure.
<u>relation</u>	A correspondence between two sets of numbers.
<u>rotation</u> (also called a turn)	A transformation which turns a figure about a point by a given number of degrees.
<u>sample space</u>	The portion of a population from which data is drawn.
<u>statistics</u>	The branch of mathematics which is the study of the methods of collecting and analyzing data.
<u>symbolic manipulator</u>	Technological tool (graphing calculator, computer) that performs traditional algebraic tasks, such as changing the form of expressions (e.g., factoring) and solving equations and inequalities.

<u>tolerance</u>	The tolerance of a measurement is the largest possible error, generally half of the unit of measure.
<u>transformation</u>	The process of changing one configuration or expression into another in accordance with a rule. Common geometric transformations include translations, rotations, and reflections.
translation (also called a slide)	A transformation that moves a geometric figure by sliding. Each of the points of the geometric figure moves the same distance in the same direction.
<u>Venn diagrams</u>	A method of illustrating sets and their properties using overlapping and non-overlapping circles and other plane geometric figures.

SECTION VI: REFERENCES AND RESOURCES

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SECTION VII: ACKNOWLEDGMENTS

Louisiana Department of Education Project Staff

Rebecca Christian
Project Co-Director

Janie Johnson
Assessment Coordinator

William Miller
Project Co-Director

Thelda Adams
Curriculum Coordinator

Cindy DeLaughter
Project Secretary

Louisiana Systemic Initiatives Program Staff

Richard Anderson
Senior Advisor

Stephanie Williamson
Assistant Director for Mathematics

Mic Lang
Associate Director for Testing
and Evaluation

Claudia Townsend
Mathematics Coordinator

Framework Writers

Thelda Adams, Teacher
Livingston Parish

Bradd Clark, Assistant Professor
University of Southwestern
Louisiana

Karen Aguilard, Teacher
Lafayette Parish

Phyllis Desormeaux, Teacher
Lafayette Diocese

Candi Bagley, Teacher
Bossier Parish

Mary Lou Hughes, Teacher
Bossier Parish

Kathleen Chadick, Teacher
East Baton Rouge Parish

Mathematics Committee Members

**Ann Allred, Teacher
Bossier Parish**

**Pennie Childs, Teacher
Livingston Parish**

**Gwen Barsley
Bureau of Secondary Education, LDE**

**Fen Chou
Bureau of Pupil Accountability, LDE**

**Ginnie Bolin
Supervisor of Instructional Services
East Baton Rouge Parish**

**Jean Clement
Bureau of Secondary Education, LDE**

**Linda Bosch, Teacher
East Baton Rouge Parish**

**Ann Clouatre
Office of Special Education, LDE**

**Patrick Bouillion
Teacher, Lafayette Parish**

**Wilfred Parl Cluse, Teacher
St. Martin Parish**

**Michelle Bourgeois, Teacher
East Baton Rouge Parish**

**Beverly Cole, Teacher
Caddo Parish**

**Jo Boutte, Teacher
East Baton Rouge Parish**

**Lovenia DeConge-Watson
Department of Mathematics
Southern University**

**Connie Buck
Bureau of Secondary
Vocational Education, LDE**

**Debbie Dees, Teacher
Ouachita Parish**

**Donna Bucko, Teacher
Ouachita Parish**

**Debra Dixon
Bureau of Program Development,
LDE**

**Janice Carter, Teacher
Orleans Parish**

**Gervis Duhon, Title 1 Supervisor
Lafayette Parish**

**Stan Chadick
Mathematics Department
Northwestern State University**

**Alice Efferson, Teacher
East Baton Rouge Parish**

**Theresa Champagne, Teacher
Bossier Parish**

**Velma Fuller, Teacher
Caddo Parish**

**Brenda Gates, Teacher
Livingston Parish**

**Clara Glasper
Supervisor of Science
East Baton Rouge Parish**

**Cathy Glover, Teacher
East Baton Rouge Parish**

**Sharon Grand, President
Louisiana Association of Teachers
of Mathematics**

**Pam Haley
Mathematics Coordinator
Bossier Parish**

**Zoe Harrell, Teacher
East Baton Rouge Parish**

**Octavio Hernandez, Teacher
Jefferson Parish**

**Mary Hitt, Teacher
St. Charles Parish**

**Jan Jarrell, Supervisor
Jefferson Parish**

**Susan Johnson
Bureau of Elementary Education, LDE**

**Willie Johnson
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Baton Rouge Chamber of Commerce**

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Southern University**

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Mathematics Department
University of Southwestern Louisiana**

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Bureau of Elementary Education, LDE**

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Child Welfare and Attendance
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LaSIP Site Coordinator
Northwestern State University**

**Mary Lou Jumonville
LaSIP Site Coordinator
University of Southwestern Louisiana**

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St. Martin Parish**

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St. Joseph Academy**

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Bureau of Secondary Education, LDE**

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Louisiana State University**

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Bureau of Secondary Vocational
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Mathematics Department
Louisiana State University**

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Mathematics Department
Louisiana State University-Eunice**

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Nicholls State University**

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Orleans Parish**

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Department of Mathematics
Dillard University**

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Terrebonne Parish**

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Bienville Parish**

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Bossier Parish**

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East Baton Rouge Parish**

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Webster Parish**

**Delores Simoneaux,
Assistant Professor
Southeastern Louisiana University**

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Vernon Parish**

**Ellen Watson, Teacher
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Iberville Parish

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Franklin Parish

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Bossier Parish

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Lincoln Parish

Beverly Blanchard
Ascension Parish

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Bienville Parish

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Jefferson Parish

Alisa Antley
Lincoln Parish

Karla Booth
Tangipahoa Parish

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Evangeline Parish

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Lafourche Parish

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Washington Parish**

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Tangipahoa Parish**

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Pointe Coupee Parish**

**Gerald Sandidge
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**Donna Myers
West Feliciana Parish**

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Claiborne Parish**

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Charlene Serrette

Bossier Parish

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Lafayette Parish**

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St. Landry Parish**

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Monroe City Schools**

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Vernon Parish**

**Stella Williams
Orleans Parish**

**James Williams
Calcasieu Parish**

**Daneen Witty
West Baton Rouge Parish**

Field Review Facilitators

**Candi Bagley, Teacher
Bossier Parish**

**Nora Miller, Teacher
East Baton Rouge Parish**

**Jo Boutté, Teacher
East Baton Rouge Parish**

**Kay Neuse
LaSIP Regional Coordinator
Region VII**

**Pam Haley, Mathematics Coordinator
Bossier Parish**

**Ray Robicheaux
Mathematics Department
Louisiana State University-Eunice**

**May Lou Hughes, Teacher
Bossier Parish**

**Ellen Watson
LaSIP Regional Coordinator
Region VI**

**Mary Lou Jumonville
LaSIP Site Coordinator
University of Southwestern Louisiana**

Year One Monitoring Team

**Gayle Potter
Arkansas Department of Education**

**Cynthia Ward
Mississippi Department of Education**

**Catherine Wilcoxson
Nebraska Department of Education**